P142: Icarus II

File Version: 1.0

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April 9th, 2014

Useful Rocket Equations

1 Introduction

This document is intended to be a reference guide for those involved in Rocketry, either amateur rocket building, creating computer simulations, games, or just those who wish to learn. This reference is by no mean exhaustive, it is simply a record of the equations that I found useful (and essential) while working on "Icarus", my simulation program and rocket design.

The vast majority of these equations have been sourced from various web resources, some of which were very hard to find. I have also added some simple equations of my own that I used for my project. Due to the significant amount of time I spent finding and learning how to use these equations, I thought it would be useful to release this resource to the community, in the hope that it could help others in the future.

While I have taken care to check these equations for accuracy, I make no guarantee to this effect, and accept no responsibility for any errors.

I have chosen to use mathematical notation throughout the document, for consistency and professionalism. (eg. ρ instead of "rho"). This is despite the fact that I originally used them in a computer program.

Contents

2 Mathematical Symbols

The mathematical symbols used in this document are as follows:

Δ = Delta

π = Pi

γ = gamma

κ = kappa

$\dot{m}$ = m Dot

ζ = zeta

ρ = rho

ln = Natural Log

R' = R Dash

$L*$ = L Star

3 Delta (Δ)v: (Tsiolkovsky rocket equation) [1]

$\Delta v = ve\ \ln\left(\frac{m0}{m1}\right)$

where:

Δv = Delta-v, The maximum change in velocity (of the vehicle)

m0 = Wet Mass (Total weight of rocket, including fuel)

m1 = Dry Mass (Weight of Rocket excluding fuel)

ve = Exhaust Velocity

ln = Natural Logarithm

4 Exhaust Velocity:[2]

$$Ve = \sqrt{\left(\frac{2\kappa}{\kappa-1}\right)\left(\frac{RTc}{M}\right)\left(1-\left(\frac{Pe}{Pc}\right)^{\frac{(\kappa-1)}{\kappa}}\right)}$$

where:

κ = Specific heat ratio (Heat Capacity Ratio,

Isentropic Expansion Factor)

$R*$ = Universal gas constant (8,314.4621 J/kmol-K in SI units)

Tc = Combustion temperature

M = Average molecular weight of the exhaust gases

Pc = Combustion chamber pressure

Pe = Nozzle exit pressure

5 Exhaust Velocity: (Alternate)[3]

$$Ve = \sqrt{\left(\frac{TR}{M}\right)\left(\frac{2\gamma}{\gamma-1}\right)\left(1-\left(\frac{Pe}{P}\right)^{\frac{(\gamma-1)}{\gamma}}\right)}$$

where:

ve = Exhaust Velocity at nozzle exit, m/s

T = Absolute temperature of inlet gas, K

R = Universal gas law constant = 8314.5 J/(kmol-K)

M = Gas molecular mass, kg/kmol

$\gamma = \frac{Cp}{Cv}$ = Isentropic expansion factor (ratio of specific heats)

Cp = Specific heat of the gas at constant pressure

Cv = Specific heat of the gas at constant volume

Pe = Absolute pressure of exhaust gas at nozzle exit, Pa

p = Absolute pressure of inlet gas, Pa

6 Exhaust Velocity: (Alternate 2)[4]

$$Ve^2 = \frac{\kappa Rgas Tc[1-(\frac{pe}{pc})^{\frac{(\kappa-1)}{\kappa}}]}{(\kappa-1)}$$

where:

κ = Ratio of specific heats, $\frac{cp}{cv}$

pe = Nozzle exit pressure

pc = Combustion chamber pressure

Tc = Combustion chamber temperature

$Rgas$ = Exhaust flow specific gas constant ($\frac{R}{M}$)

R = Universal gas constant

M = Exhaust gas molecular weight

7 Effective Exhaust Velocity (when Specific Impulse is known:)[5]

$Ve = g0Isp$

where:

Isp = Specific impulse in seconds

ve = Specific impulse measured in m/s, which is the same as the effective exhaust velocity measured in m/s

$g0$ = The acceleration due to gravity at the Earth's surface, $9.81 m/s^2$

8 Specific Impulse when exhaust velocity is known:[5]

Isp = $\left(\frac{ve}{g0}\right)$

where:

Isp = The specific impulse in seconds

ve = Average exhaust speed along the axis of the engine (ft/s or m/s)

$g0$ = The acceleration due to gravity at the Earth's surface, $9.81 m/s^2$

(in ft/s^2 or m/s^2)

9 Specific Impulse given Thrust and Mass Flow Rate:[6]

$Isp = \left(\frac{F}{\dot{m} g_0}\right)$

where:

Isp = The Specific Impulse

F = Thrust

$\dot{m}$ = Exit Mass Flow Rate

g_0 = Gravitational Acceleration Constant ($9.8 m/sec^2$)

10 Gravity Losses. Assume Rockets flight is vertical.[7]

$Lg = gt$;

where:

Lg = Gravity loss of velocity

g = Gravitational acceleration

t = Duration of rocket thrust

11 Thrust:[9]

Thrust $= v\frac{dm}{dt}$

where:

T = Thrust Generated (force)

v = Speed of Exhaust Gases (Relative to Rocket)

$\frac{dm}{dt}$ = Mass Flow Rate of Exhaust (rate of change of mass with respect to time)

Force = Thrust

11.1 Newton's Second Law:[18]

$f = ma$

where:

f = Force

m = Mass

a = Acceleration

11.2 Impulse:[8]

I $= Faverage\Delta T$;

where:

I = Impulse

$Faverage$ = Average Force

ΔT = Time

11.3 Impulse:[8]

Force = Thrust

I = $m\Delta v$;

where:

I = Impulse

m = Mass

Δv = Change in Velocity

12 Burn Calculations:

12.1 Burn Time:

$BurnTime = \frac{FuelQuantity(mass)}{exhaustRate}$

12.2 Burn Rate:

$BurnRate = \left(\frac{mass}{exhaustvelocity}\right)$

13 Drag:[10][11]

$D = \frac{1}{2}\rho v^2 CdA$ or:

$D = \left(\frac{cdA\rho v^2}{2}\right)$

Where:

D = Fd = Drag(Newtons)

ρ = Mass density of the fluid

v = Velocity of the object relative to the fluid

Cd = Drag Coefficient

A = Reference area

14 Volume/Mass/Density:[12]

$V = \frac{m}{\rho}$

$\rho = \frac{m}{V}$

$m = \rho V$

where:

ρ = Density

m = Mass

V = Volume

14.1 Total Density:

$TotalDensity = (FuelDensity + OxidiserDensity)$

15 Dimensions of a Cylinder: (For Fuel/Oxidiser Tanks)[13]

15.1 Volume of Fuel/Oxidiser Tank:

$V = \pi R^2 L$

where:

V = Volume

π = Pi

R = Radius

L = Length

15.2 Length of Fuel/Oxidiser Tank:

$L = \left(\frac{V}{\pi R^2}\right)$

where:

V = Volume

π = Pi

R = Radius

L = Length

16 Nose Cone Equations:

16.1 Volume (Parabolic cone):[14]

$V = \left(\frac{2\pi d^2 h}{15}\right)$

where:

V = Volume

π = Pi

d = Diameter

h = Height

16.2 Height (Parabolic cone):[14]

$H = \left(\frac{15V}{2\pi d^2}\right)$

where:

H = Height

π = Pi

d = Diameter

V = Volume

16.3 Formula for Von Karman Nose Cone:[15]

$Radius = 5\sqrt{\frac{\arccos\left(1-2\left(\frac{x}{25}\right)\right)-0.5\sin\left(2\arccos\left(1-2\left(\frac{x}{25}\right)\right)\right)}{\pi}}$

where:

Diameter $= 5$

17 Propellant Mass Fraction[16]

$\zeta = 1 - \left(\frac{dM}{wM}\right)$

hence:

$wM = \left(\frac{dM}{\zeta}\right)$

$dm = wM - (wM\zeta)$

where:

ζ = (zeta) Propellant Mass Fraction

dM = Dry Mass (m_f, final mass)

wM = Wet Mass (m_0, initial mass)

18 Dynamic Pressure: [17]

$q = \left(\frac{1}{2}\right)\rho v^2$

where:

q = Dynamic pressure (in pascals)

ρ = Fluid density in kg/m^3

v = Fluid velocity in m/s

and:

$q = \left(\frac{1}{2}\right)\rho PsM^2$

where:

q = Dynamic pressure (in pascals)

Ps = Static pressure (pascals)

M = Mach number

ρ = Ratio of specific heats

Static pressure = Exhaust pressure

19 Acceleration[17]

$a = \left(\frac{\Delta v}{T}\right)$

where:

a = Acceleration

$\Delta\ v$ = Change in velocity (Δv = Final velocity - Initial velocity)

T = Time

or, from Newton's Second Law: [18]

$a = \left(\frac{f}{m}\right)$

where:

a = Acceleration

f = Force

m = Mass $speed = \left(\frac{distance}{time}\right)$

20 Equations to Determine Dimensions of Rocket Nozzle:[2][19]

20.1 Diameter of the Throat:

$Dt = \sqrt{4\left(\frac{At}{\pi}\right)}$

where:

Dt = Nozzle Throat Diameter

At = Nozzle Throat Area

π = Pi

20.2 Diameter of the Exhaust:

$De = \sqrt{\frac{4Ae}{PI}}$

where:

De = Nozzle Exhaust Diameter

Ae = Nozzle Exhaust Area

π = Pi

20.3 Combustion Chamber Diameter:

Dc = least 5Dt (Five times the Nozzle throat diameter)

where:

Dc = Combustion Chamber Diameter

Dt = Nozzle Throat Diameter

20.4 Area of the Throat:[20]

$At = \frac{wt}{pt}\sqrt{\frac{RTt}{\gamma gc}}$

where:

At = Nozzle Throat Area

wt = Propellant flow rate at throat

pt = Pressure at throat

R = Gas Constant (see below)

Tt = Temperature of Gases at nozzle throat

gc = Gravitational constant ($9.81 * 10^-1$)

γ = Specific heat ratio (Isentropic expansion factor)

20.4.1 R:

$R = \frac{R\_bar}{M}$

where:

R_bar = Universal gas constant (8.3144621 $\frac{J}{molK}$)

M = Molecular weight of the gas

20.5 Area of the Throat: (metric)

$At = \frac{q}{Pt}\sqrt{\frac{R'Tt}{Mk}}$

where:

At = Nozzle throat Area

q = Propellant mass flow rate (kg/s)

R' = Universal gas constant (8.3144621$\frac{J}{molK}$

M = Molecular weights of gas (kg/mol)

γ = Specific heat ratio (Isentropic expansion factor)

20.6 Area of the Nozzle Exit:

$Ae = \left(\frac{At}{Nm}\right)\left[\frac{1+\left(\frac{\gamma-1}{2}\right)Nm^2}{\frac{(\gamma+1)}{2}}\right]^{\left(\frac{\gamma+1}{2(\gamma-1)}\right)}$

where:

Ae = Nozzle exit area

At = Nozzle throat area

Nm = Mach Number (See below)

γ = Specific heat ratio (Isentropic expansion factor)

20.6.1 Mach Number:

$Nm2 = (2/(\gamma - 1)[(\frac{Pc}{Pa})^{\frac{(\gamma-1)}{\gamma-1}}$

where:

Nm = Mach Number

Pc = Combustion chamber pressure (pascals)

Pa = Ambient atmospheric pressure

20.7 Gas temperature at the Nozzle throat:

$Tt = Tc/(1 + (\gamma - 1)/2)$

and:[21]

$Tt = Tc[1/(1 + (\gamma - 1)/2)]$

where:

Tt = Gas temperature of the nozzle throat

Tc = Flame temperature of combustion chamber (Kelvin)

γ = Specific heat ratio (Isentropic expansion factor)

20.8 Gas pressure at nozzle throat:

$Pt = Pc[1 + \frac{(\gamma-1)}{2}] - \frac{\gamma}{(\gamma-1)}$

where:

Pt = Gas pressure at nozzle throat

Pc = Combustion chamber pressure (pascals)

γ = Specific heat ratio (Isentropic expansion factor)

20.9 Length of Combustion Chamber:

$Lc = \frac{Vc}{Ac}$

where:

Lc = Length of Combustion Chamber

Vc = Volume of Combustion Chamber

Ac = Area of Combustion Chamber

20.10 Volume of the Combustion Chamber:

$Vc = 1.1(AcLc)$

where:

Vc = Volume of Combustion Chamber

Ac = Area of Combustion Chamber

Lc = Length of Combustion Chamber

and:

$Vc = LAt$

where:

Vc = Volume of Combustion Chamber

$L*$ = (L Star) Characteristic Length (See below)

At = Area of the throat

20.10.1 Characteristic Length:

$L* = \frac{Vc}{At}$

where:

$L*$ = (L Star) Characteristic Length

Vc = Volume of Combustion Chamber

At = Area of the throat

20.11 Area of the Combustion Chamber:

$Ac = \frac{\pi Dc^2}{4}$

where:

Ac = Combustion Chamber Area

π = Pi

Dc = Combustion Chamber Diameter

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